Grade 7 Mathematics, Quarter 2, Unit 2.1

Recognizing Proportional Relationships and Finding Unit Rates

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Compute unit rates associated with ratios of fractions.
- Compute unit rates associated with like and unlike units.
- Recognize proportional relationships in tables.
- Recognize and represent proportional relationships in equations.
- Recognize proportional relationships in graphs/diagrams.
- Recognize proportional relationships in verbal descriptions.
- Test for equivalence in tables and graphs.
- Recognize proportionality in lines that are straight through the origin
- Explain what the coordinates (x, y) represent in terms of the situation specifically involving unit rate.

Mathematical practices to be integrated

Attend to precision.

- Specify units and label axes.
- Use units in discussions about their reasoning.

Look for and make use of structure.

- Look for patterns to determine whether a relationship between two quantities is proportional.

Look for and express regularity in repeated reasoning.

- Use proportional reasoning to solve problems with missing quantities.
- See slope as the unit rate.

Essential questions

- How do you find unit rate when working with fractions?
- How do you find unit rate when the units are different?
- How do you determine if the quantities in a table have a proportional relationship?
- How do you determine if the coordinate pairs on a graph are proportional?
- How do you determine if an equation represents a proportional relationship?
- What does a straight line through the origin tell you about the points along the line?

Written Curriculum

Common Core State Standards for Mathematical Content

Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, ~~areas~~ and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction $^{1/2}/_{1/4}$ miles per hour, equivalently 2 miles per hour.*

7.RP.2 Recognize and represent proportional relationships between quantities.

a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.*

d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.

Common Core State Standards for Mathematical Practice

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

Sixth-grade students learned about the concept of a ratio and used language including unit rate. They plotted points and discussed ratio relationships between two quantities in a real-world context. Students used tables and equivalence to find missing values.

Current Learning

Students compute unit rates, including ratios of fractions in like or different units; this is a critical area for reinforcement. Students observe whether a graph is a straight line through the origin. They identify and represent constant unit rates in tables, graphs, equations, and verbal descriptions by testing for equivalence in tables and graphs. Students explain the meaning of coordinate pairs in terms of the situation being represented.

Future Learning

In Grade 8, students will use proportional reasoning to define, evaluate, and compare functions. They will also solve and analyze linear equations and pairs of simultaneous linear equations. Students will use their understanding of proportional relationships to make connections between lines and linear equations.

Additional Findings

In *Adding it Up: Helping Children Learn Mathematics,* the authors state that proportional reasoning is the "gateway to higher mathematics." (p. 242)

According to *Principles and Standards for School Mathematics*, "facility with proportionality is more than setting two ratios equal to each other . . . it involves thinking about two quantities and their relationship." (p. 217)

Grade 7 Mathematics, Quarter 2, Unit 2.2

Evaluating Expressions and Solving Equations

Overview

Number of instructional days: 11 (1 day = 45–60 minutes)

Content to be learned

- Use order of operations to factor and expand linear expressions with rational coefficients.
- Generate equivalent expressions by rewriting expressions in different forms to show relationships between quantities.
- Fluently solve multistep numerical and algebraic expressions using rational numbers in any form.
- Fluently apply properties of operations to calculate multistep numeric and algebraic expressions with rational numbers in any form.
- Fluently apply properties of operations to assess the reasonableness of answers to multistep problems by using mental mathematics and estimation.
- Fluently solve word problems leading to equations in the form $px + q = r$, which are rational numbers.
- Fluently compare the steps used in arithmetic solutions to the steps used in algebraic solutions.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Make use of quantities and their relationships in problem situations.
- Create coherent representations of the problem, flexibly using different properties of operations and objects.

Look for and make use of structure.

- View complicated algebraic expressions as problems composed of simpler arithmetic operations.

Look for and express regularity in repeated reasoning.

- See and make use of repeated quantities to factor and expand linear expressions.

Essential questions

- How do you use order of operations when factoring linear expressions?
- How do you use order of operations when expanding linear expressions?
- How can you use equivalent expressions to show how quantities in different forms are related?
- How do you use algebraic expressions to solve multistep problems that include numbers in any form?
- Why are the properties of operations necessary when calculating multistep problems involving a variable?
- What are two ways you can assess the reasonableness of your answers to multistep problems?
- What word problem could you write that would lead to an equation in the form of $px + q = r$?
- How do the steps used in solving arithmetic problems compare to the steps used in algebraic problems?

Written Curriculum

Common Core State Standards for Mathematical Content

Expressions and Equations 7.EE

Use properties of operations to generate equivalent expressions.

7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

7.EE.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."*

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*

7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

Common Core State Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.Clarifying the Standards

Prior Learning

In sixth grade, students learned how to read, write, evaluate, and solve expressions and equations in which letters stand in for numbers. They applied the properties of operations to generate equivalent expressions. Students solved real-world problems by writing and solving equations in the form of $p + x = q$.

Current Learning

Students apply the properties of operations to factor and expand linear expressions with rational coefficients and understand that rewriting an expression in different forms can shed light on the problem and how quantities are related. Students fluently use multistep problems with rational numbers in any form, converting between forms and assessing reasonableness of answers based on estimation and mental mathematics to solve real-life mathematical problems. Students also graph and solve equations fluently in the form of $px + q = r$; where p, q, and r are specific rational numbers.

Future Learning

In Grade 8, students will use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. They will also solve and analyze linear equations and pairs of simultaneous linear equations, recognizing the constant of proportionality as slope (m).

Additional Findings

In *Adding it Up: Helping Children Learn Mathematics,* the authors state that the shift from arithmetic to algebra "requires thinking that proceeds in rather different ways from the thinking that develops in traditional arithmetic . . . this requires many adjustments, even for those students who are quite proficient in arithmetic." (p. 261)

According to *Principles and Standards for School Mathematics*, "in middle grades, students should learn to recognize and generate equivalent expressions, solve linear equations, and use simple formulas." (p. 223)

Grade 7 Mathematics, Quarter 2, Unit 2.3

Solving and Graphing Inequalities

Overview

Number of instructional days: 5 (1 day = 45–60 minutes)

Content to be learned

- Solve real-life word problems involving inequalities.
- Solve real-life inequalities in the form of $px + q > r$ or $px + q < r$.
- Graph solutions sets for inequalities.
- Interpret the solutions in a real-world context.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Interpret solutions in real-world situations.
- Attend to the meaning of quantities.

Attend to precision.

- Solve inequalities accurately and communicate the meaning of solutions.
- Label axes appropriately.

Look for and express regularity in repeated reasoning.

- Evaluate the reasonableness of solutions.

Essential questions

- How do inequalities appear in the real world?
- How do you solve inequalities?
- How do you graph inequalities?
- What does the solution to your inequality tell you?

Written Curriculum

Common Core State Standards for Mathematical Content

Expressions and Equations	7.EE

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*

Common Core State Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the

regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In sixth grade, students learned how to read, write, and evaluate expressions, and they solved equations in which variables stand in for numbers. They applied the properties of operations to generate equivalent expressions. They have also written and graphed inequalities on a number line and reasoned about the infinite solutions within a real-world context.

Current Learning

Students graph and solve equations and inequalities fluently in the form of $px + q = r$; where p, q, and r are specific rational numbers. Students also interpret the solution to their inequalities in a real-world context.

Future Learning

In Grade 8, students will use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. They will also solve and analyze linear equations and pairs of simultaneous linear equations, recognizing the constant of proportionality as slope (m).

Additional Findings

In the PARCC 6-8 Expressions and Equations progression, the authors state that "students set up and solve inequalities . . . recognizing that the process is similar to solving linear equations." (p. 9)

Grade 7 Mathematics, Quarter 2, Unit 2.4

Analyzing Proportional Relationships

Overview

Number of instructional days: 9 (1 day = 45–60 minutes)

Content to be learned

- Solve and analyze a multistep real-world ratio problem using proportional reasoning**.**
- Solve and analyze a multistep real-world percent problem using proportional reasoning.

Mathematical practices to be integrated

Construct viable arguments and critique the reasoning of others.

- Use proportional reasoning to defend the final price of an item.
- Use proportional reasoning to argue the discounted price of an item.

Look for and make use of structure.

- Use patterns or structures to solve multistep percent problems.

Look for and express regularity in repeated reasoning.

- Evaluate the reasonableness of their responses to multistep ratio and percent problems.

Essential questions

- When given a price and a salex tax rate, how do you find the final price?
- When given a price and a markdown, how do you find the final price?
- When given your bill at a restaurant, how do find the amount needed for gratuity? How do you find your total bill including gratuity?
- If given a new price after a markdown, how do you find the original price?
- If given an original price and markup, how do you find the new price?

Written Curriculum

Common Core State Standards for Mathematical Content

Ratios and Proportional Relationships 7.RP

Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

Common Core State Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2)

with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In sixth grade, students learned how to find the percent of a quantity as a rate per 100. Sixth-grade students used and understood the language of ratios to understand the concept of a ratio as a relationship between two quantities.

Current Learning

Students are in the development and reinforcement stages of analyzing and using percent and ratio to solve multistep real-world problems, including but not limited to applications such as tax, discount, and percent increase or decrease.

Future Learning

In Grade 8, students will graph proportional relationships and interpret the unit rate as the slope (m) of a line.

Additional Findings

In the PARCC 6-7 Ratios and Proportional Relationships progression, the authors state that "students extend their work to solving multi-step ratio and percent problems . . . requiring careful attention to the referent whole." (p. 10)

According to *Principles and Standards for School Mathematics*, "proportionality is an integrative thread that connects many mathematics topics studied in 6th through 8th grade." (p. 217)